Mean and Variance of Discrete Random Variables

Previously, we summarized a list of numbers by computing their average and SD. Now we'll do the analogous summaries for random variables (#’s generated by a chance process).

The mean of a random variable is also known as the **expected value** (EV). The expected value of a discrete random variable $X$ is shown by:

\[
\mu_X = E(X) = X_1P_1 + ... + X_nP_n
\]

...where $X_i$: Value of event #i

$P_i$: Probability of event #i

The standard deviation of a random variable as the **standard error** (SE) and measures the spread. The SE of a discrete random variable $X$ is shown by:

\[
\sigma_X = \sqrt{(X_1 - \mu_X)^2P_1 + ... + (X_n - \mu_X)^2P_n}
\]

...where $X_i$: Value of event #i

$\mu_X$: Expected value of $X$ (see left)

$P_i$: Probability of event #i

Puzzle #1: Let’s say $X$ is a random variable that looks at the number of workouts that I will do in a week. Here’s the distribution. Find the expected value (EV) of $X$ and the standard error (SE) of $X$:

<table>
<thead>
<tr>
<th>$X$</th>
<th>$P(X)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>1</td>
<td>0.15</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
</tr>
</tbody>
</table>

We can also make a histogram of all possible outcomes of a chance process and their probabilities. **This type of histogram is known as a probability histogram.** Probability histograms for discrete random variables are also known as probability mass functions (**pmf**). Probability histograms for continuous random variables are also known as probability density functions (**pdf**). Draw the pmf below:

**Probability Histogram drawn by hand:**

**Probability Histogram from Python**